PROFILING THE MAGNETIC FIELD OF THE NEW POLE TIPS ON THE RUTGERS 12-INCH CYCLOTRON

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Abstract

The preliminary design of the Rutgers 12-inch cyclotron did not account for the axial velocity components of protons as they are produced near the ion source. These axial velocities lead to a loss in beam luminosity due to collisions with the chamber floor and ceiling as protons travel along their orbits. To correct for this undesired effect, an adjustment was made to the shape of the initially flat and featureless ends of the magnets (pole tips) used in the cyclotron. This adjustment is intended to generate radial, or nonaxial, components to the magnetic field that induces axial oscillations of the protons, which dampens near the edge of the chamber producing a focused beam. We profile the magnetic field of the new pole tips to verify that the adjustments are indeed appropriate to produce the desired beam focusing and calculate the n-value of the magnets as a function of chamber radius.

1. INTRODUCTION

1.1. BACKGROUND

Cyclotrons were among the very first particle accelerators ever produced. Their theoretical simplicity made them relatively easy to build and operate. In fact, the inventor of the cyclotron, Ernest O. Lawrence, constructed his first cyclotron from coffee cans, sealing wax, and leftover lab equipment¹. The premise of a cyclotron is the same as that of a linear accelerator, to accelerate charged particles through a sequence of potential differences that boost the particle's velocity in steps. The ingenuity in cyclotron design is the confinement of this sequence to a small space by winding the particles' trajectory into nested circular orbits of increasing radius. In this design, the size of a cyclotron is just that of the particles' greatest orbit versus the size of a linear accelerator, which depends on the number of potential steps used. This makes it possible for cyclotrons to be used in the confines of a lab unlike linear accelerators.

1.2. THEORY AND CYCLOTRON DESIGN

In order to wind the particles' trajectory and confine it to the cyclotron chamber, one utilizes magnetic fields to curve their path, in conjunction with the electric potentials that increase the particles' velocity. A charged particle moving through a magnetic field experiences a force proportional to its velocity and the strength of the magnetic field. The magnitude and direction of the force is given by,

$$\vec{F} = q\vec{v} \times \vec{B}$$

thus the force is always perpendicular to the plane determined by the direction of the charged particle's motion and the direction of the magnetic field. In the case of uniform magnetic fields, as found in the simplest cyclotron design, the resulting particle trajectories are either circular or helical with small pitch. The radius of a particle orbit at any given time in its trajectory is thus,

$$ma = m \frac{v_{\perp}^2}{r} = qv_{t}B \implies r = \frac{mv_{\perp}}{qB}$$

where v_{\perp} is the particle's velocity component perpendicular to the direction of the magnetic field. Clearly, as the velocity of the particle in this direction increases, its orbital radius will increase also. In this way a particle starting to move near the center of the chamber, will move in a spiral outwards to the edge of the chamber.

The cyclotron chamber, where the particle acceleration is accomplished, contains two separated charged metal cavities or "Dees", so called because of their shape, held at different electric potentials. The dees (fig 1.) are two half-circle metal plates, each held together by a curved metal wall, basically two halves of a hollowed-out disc. The potential difference between the dees serves to accelerate the charged particles as they pass through the gap between the dees. Charged particles are initially introduced to the chamber inside this gap, where they are immediately accelerated by the potential and enter the dees' cavity. Inside the cavity, particles are electrically isolated and only feel the presence of a magnetic field. The uniform magnetic field curves the path of the particle into a circular orbit, directing it back towards the gap. At this point, the electric field inside the gap must flip direction or the particle will be accelerated backwards and will loose the velocity it gained thus far. It is clear that the electric field flip must occur periodically at a rate that is dependent on the time it takes for a particle to complete half of an orbit inside one of the cavities, otherwise it will re-enter the gap and be repelled backwards. It is an extraordinary fact that the rate of this flip in the electric field is independent of the velocity of the particle. The time of one orbit of an object in circular motion is given by $T = 2\pi r/v$, but we found how the radius of orbit is dependent on the velocity above, so that

$$T = \frac{2\pi r}{v_{\perp}} = 2\pi \frac{m}{qB} \Longrightarrow \omega_0 = \frac{qB}{m}$$

where ω_0 is the resonant frequency of the cyclotron. This resonant frequency, or resonant condition between the magnetic field and the electric field flipping, guarantees that as the particle moves in its circular orbit within the cavities of the dees it returns to an electric field that accelerates, rather than retards, its motion.

Thus the cyclotron functions as follows. Charged particles are introduced by an ion source into the gap between the charged dees. The particles are accelerated through the potential, enter the cavities and move in a circular path determined by their velocity and the strength of the magnetic field. By the time they return to the gap, the charge on the dees is reversed and the electric field between them flips so that the particle is again accelerated. As this process repeats, the velocity of the particle increases with each half turn through the chamber, and the radius of its orbit increases proportionally until it reaches a point where it either collides with the walls of the dees or is directed out of the chamber. Of course this is a simplified model of a cyclotron that does not take many real world obstacles into account such as non-uniform magnetic fields, divergent particle beams (stream of accelerated charged particles that the cyclotron produces), loss of charged particles to collisions, and challenges in producing the charged particles to begin with. The Rutgers 12-inch cyclotron faces many of these obstacles, particularly the divergent particle beam that is closely linked to its charged particle production mechanism (ion source).

In the Rutgers 12-inch cyclotron, the charged particles being used are protons that are produced by ionizing hydrogen gas in the chamber. A diffusion pump is constantly evacuating the chamber while hydrogen gas is simultaneously being introduced to the chamber for ionization, producing a stream of hydrogen gas inside the dee gap. A high temperature, negatively charged cathode is placed at the mouth of the stream and thermally emits electrons that are then repelled from the negatively charged cathode. The initial velocity of the electrons causes them to spiral very tightly into a helical trajectory along the magnetic field. This creates an electron beam that is directed at the grounded chamber floor. When this electron beam collides with the hydrogen gas stream, the hydrogen is ionized and protons are freed inside the dee gap. While this method is an elegant and simple way of generating a source of charged particles, the particles produced have an essentially random distribution of velocities. This means that a large fraction of your particles will follow helical trajectories as they propagate outwards in the chamber. If this is the case, many of them will collide with the dees well before the reach the end of the chamber. This is an important effect that may not be neglected and must be corrected for. It is important for a well-manufactured cyclotron be efficient, without significant loss beam luminosity, in producing a focused beam that may be directed out of the chamber for use in experiments.

Besides the ion source, there are other noteworthy components to the Rutgers cyclotron. The magnetic field of the Rutgers cyclotron is produced from two large 1500-pound cylindrical iron electromagnets powered by two 800-pound coils. The chamber, a roughly 2-inch thick disc, is sandwiched between the tips of the magnets (the pole tips) and is of course, 12 inches in diameter, as are the magnets. The important difference between the Rutgers 12-inch cyclotron and the basic model is that the Rutgers cyclotron replaces the second dee with a "dummy". The dummy dee uses the chamber walls instead of a separate piece of metal as the cavity for the particles to move in. In this model, the first dee is charged while the rest of the chamber is grounded. This simplifies the process of flipping the charges on both of the dees to reversing the charge on only one. This also provides for a simpler means of bringing the particle beam out of the chamber, without needing to pass it through or around a dee first.

2. EXPERIMENT AND DATA

2.1. EXPERIMENTAL MOTIVATION

The major accomplishment in the production of a cyclotron is the creation of an ion beam. Once the beam is made, it is of utmost importance that it have the properties that make it effective in experiments in which it is intended to be used. In order to accomplish both initial beam creation and subsequent beam shaping, it is essential that the cyclotron contain a beam focusing mechanism. The most common, and perhaps most efficient, beam focusing mechanism is a magnetic field shaped such that a particle wandering off of the ideal orbit is met with a restoring force, thus creating a beam of charged particles oscillating about the ideal orbit. The original Rutgers 12-inch cyclotron had an almost perfectly uniform, axial magnetic field, resulting from its extremely flat pole pieces. Therefore, orbiting charged particles in the chamber subject to the magnetic field felt only the radial force which provides the centripetal acceleration. Any axial velocity of a particle was unaffected throughout the particle's trip, allowing it to drift up/down and potentially collide with the chamber ceiling/floor. A new magnetic field shape was needed to create the restoring forces necessary for beam focusing. A field that decreases slightly with increasing radius, so that the magnetic field lines are concave inward, produces one restoring force in the axial direction toward the median plane and another in the radial direction toward the equilibrium orbit. (The decrease in field strength must be small so as to disturb only slightly the resonance condition. If the disturbance is small enough, the orbit of the particle and the potential modulation will be out of phase by less than π over the full trip of the particle. Thus, the particle will be accelerated with each gap crossing.) Such a field can be created by shaping pole pieces so that the distance between the top and bottom pole pieces increases slightly with increasing radius. With this in mind, new pole pieces were shaped into extremely shallow cones, dropping 2% in thickness (1 in. to 0.98 in.) over a region five inches in radius about the center. In addition the edges of the pole pieces were rounded to reduce the fringing effects produced by the sharp corner of the flat pole pieces. This is not the only way to generate the desired field shape, see figure 2.

This experiment was an investigation of the magnetic field created by these new pole pieces. By measuring the profile of the magnetic field (along a radius of the pole piece) and analyzing the data obtained, we are able to estimate how effective this design is at creating a focused beam through comparison with previously successful designs and provide a means for calculating specific properties of the focusing given (as yet unknown) machine parameters.

2.2. EXPERIMENTAL METHODS

The profiling of the radial dependence of the magnetic field between the pole pieces required an experimental setup that allowed for acquisition of the magnetic field strength at precisely known positions with respect to the center of the pole pieces. In order to achieve this difficult goal, a Hall probe was mounted on a platform that was threaded onto a long screw (~12 in.) whose motion was driven by a computer-controlled stepper motor. This entire unit was set upon three adjustable "leveling" screws protruding from an aluminum base secured to the core of the magnet. An aluminum base was used because aluminum has poor magnetic properties and will not disrupt the measurements. The Hall probe was connected to a Gaussmeter whose voltage output was the input for a multimeter. The output of the multimeter was fed into a data acquisition unit, and a LabView program wrote the voltage into a text file.

Adjustments of the leveling screws allowed for leveling of the probe unit so that all measurements would be taken in the median plane. If this leveling were not possible and the measurements were taken along a plane intersecting the median plane at an angle, the axial variation in the magnitude of the magnetic field would introduce variations that mask the true radial profile. Other complications would arise from the fact that the direction of the magnetic field changes with axial displacement from the median plane (and with radius when not in the median plane) and that the Hall probe is aligned to measure the axial component of the field when it is level to the median plane but measures other components when tilted. These issues were not a concern with the unit leveled to the median plane. The radial dependence of the direction of the magnetic field off of the median plane, i.e., the probe had to be equidistant from the top and bottom pole pieces. This was accomplished with the leveling screws as well.

A LabView program (*Bfield 3.1*) provided the controls for the stepper motor and, subsequently, the Hall probe. The basic function of the program allows you to jog the probe forwards and backwards, the probe stopping only at your command. However, the program also takes as a parameter the number of stepper motor steps per inch, in other the words, the number of stepper motor steps that corresponds to a 1 in. displacement of the probe unit. In order to find this value, we mounted a dial indicator onto the aluminum base such that the

pin of the indicator was resting against the probe unit. We used the program to advance the stepper motor a known number of steps and recorded the displacement of the probe unit from the dial indicator. Dividing the number of steps by the displacement we found the number of steps per inch to be 199338.

With this parameter given, the program can be used to move the probe a specified distance rather than just jogged. We used this function to check our calibration. We ran the probe back and forth various distances and checked that they agreed with the dial indicator. More importantly, the program can also be used for taking a radial profile of the magnetic field. We specified the total distance the probe should cover (positioner length), the distance the probe should move between each measurement point (step size), and the amount of time that should elapse between the pausing of the probe and the reading of the measurement at each measurement point (wait time). Running the program moves the probe step by step and writes the measured magnetic field, step number, measured magnet current, desired magnet current, steps per inch, wait time, and step size into a text file (*BfieldMesaurements.txt*). The program also allows for the specification of the "Step Delay" which is related to the stepper motor speed. We found that the stepper motor was most reliable with a step delay of 3000.

We used this program to profile the residual field of the magnet (with the magnet off), the magnetic field produced by a solenoid with a "bullet" core with the magnet off (see Data), and the magnetic field with a magnet current of 20, 30, and 40 amps. In all measurements we used a step size of 0.05 in. and a wait time of 4 seconds. For the residual field and the bullet measurements, we set the positioner length at 4 in., while for the magnet measurements we set it at 10 in.

2.3. DATA

The first step to accurately measuring the radial profile of the magnetic field inside the magnet pole tips was to determine the actual radial dimension of the magnet. Since the hall probe unit was a separate instrument that was independent of the pole tips, we needed to carefully determine the center of the pole tips from some reference point that would tie our hall probe measurements to the pole tips. To do this we used a small bullet shaped iron solenoid core that produced a well-defined peak in our field measurements. The tip of the bullet was positioned on the hall probe unit's aluminum base at a precisely known distance of 8 inches from the center of the pole tips. Thus, when taking measurements, the center of the peak we measure at the position of the bullet corresponds to exactly 8 inches from the center of pole tips. Using the steps per inch calculations as described in the previous section, we could determine the center of the poles in our data. Alternatively, we would expect the radial profile of the tips to be peaked also, so the center of the profile peak also would provide us with the center position, but the bullet provides us with an independent method of gauging the distance.

Once the bullet position is determined relevant to the our hall probe unit's distance scale, the bullet measurement would not need to be repeated so long as the apparatus was not disturbed during subsequent runs. For this reason, as well as that the bullet physically restricted the motion of the probe unit, and the fact that the cyclotron magnets swamped the bullet field, it was not necessary to measure the bullet at the same time as the radial profile. So there was no current running through the solenoid coils at the time the bullet measurements were taken. Yet, there is a weak residual field present from the iron magnet cores even with no current, so to carefully determine the peak it was best to first profile this residual field and subtract it from the bullet profile. In figure 3 we plot two bullet profiles against the residual field, and in figure 4 we plot the residual subtracted bullet profile. The bullet profile is clearly observed as a very well defined peak. To determine the exact location of the center of the peak that should correspond with the tip of the bullet (8 inches from the pole tip center) we calculated the center-of-mass of the peak. That is we calculated a weighted average of all the data points, we now had a bullet position to within 0.025 inches, our intrinsic distance measurement error from step size. We were now prepared to begin making radial profile measurements of the pole tips.

After ironing out many wrinkles in our data acquisition VI's (with the help of Professor Kojima), we took three measurements of the radial profile starting approximately (by eye) 10 inches from the pole tip center and just under 2 inches from the bullet, and moving in past the center. Although the magnets are only 6 inches in radius, it was necessary for us to begin our measurements so far back because we needed to stay consistent with our starting point for the bullet profiles, this also allowed for enough space to measure slightly past the center and thus detect the peak in the profile. The three measurements were each taken with a different current in the solenoid coils in order to probe the profile at

different field strengths, so as to better determine the range that the cyclotron could be operated at. The three coil currents used were at 20, 30 and 40 amps. This range of current represented the allowable and safe magnet operating conditions well. Our three measurements are plotted in figures 5 and 6, we normalized each profile to the max value attained and plotted them together in figure 7, and in figure 8 we show the peaked center region of the pole tip. In figure 5 we display all the data from 9.5 inches to 0.5 inches past the center but in figure 6 we restrict the view of the profile to the actual pole tips only, we also fit lines to the central regions to demonstrate their linearity.

3. ANALYSIS AND CONCLUSIONS

A first analysis of our data is simply to check that the new pole pieces create the general magnetic field shape that they were meant to create. The design of the pole pieces was intended to produce the most effective cyclotron possible. According to Livingston and Blewett, "Experience is the best guide to the optimum magnitude of this radial decrease and the shape of the field. Those cyclotrons which have achieved high efficiency in acceleration of ions and a high-intensity beam show a consistent shape of magnetic field. This is an approximately linear decrease over most of the pole face out to the region where peripheral fringing sets in and where the decrease becomes more rapid." Figure 6 shows that the magnetic field created by the new pole pieces decreases fairly linearly from the center out to around 2.75 in. This is not as far out as would be ideal but is reasonable. Livingston and Blewett go on to state that the radial decrease from the center to the exit slit should be around 3 or 4 percent in small cyclotrons. Taking the maximum ion radius of 5 in. as the approximate location of the exit slit (there is no exit slit as of yet), we found the decrease to be about 8 percent.

In order to better discuss properties of the magnetic field relevant to focusing, it has been found useful to express the magnetic field in terms of a radial index, *n*, defined by:

$$B_z = B_0 \left(\frac{r_0}{r}\right)^n$$

where B_0 is the magnetic field at the equilibrium orbit radius, r_0 . By differentiation and substitution, one finds that

$$n = -\frac{r}{B_z} \frac{dB_z}{dr}$$

Thus, the n-value is always non-negative since the magnetic field never increases with radius. Now, using the fact that the curl of the magnetic field is zero, the radial component of a magnetic field of the above type is found to be

$$B_r = -\frac{nB_0 z}{r_0}.$$

Then, the equations of motion for a charged particle in a magnetic field become

$$\frac{d}{dt}(m\Delta\dot{r}) = -m\omega^2(1-n)\Delta t$$
$$\frac{d}{dt}(m\dot{z}) = -m\omega^2 nz$$

where ω is the angular velocity v/r₀, z is the vertical displacement from the median plane, and Δr is the radial displacement from the equilibrium orbit. Thus, a magnetic field with nvalue between 0 and 1 provides restoring forces in the radial and axial directions. So, charged particles in this field undergo oscillations about the median plane and the equilibrium orbit. The frequencies of these oscillations are related to the resonant frequency of the cyclotron, f_0 , by the n-value:

$$f_{z} = n^{\frac{1}{2}} f_{0}$$
$$f_{r} = \left(1 - n^{\frac{1}{2}}\right) f_{0}$$

Also, an increasing n-value will create a dampening of the oscillations. The larger the n-value the stronger the axial focusing, but this is limited by the fact that large deviations from a uniform field make maintaining the resonance condition difficult. For small n-values (in the region where the n-value increases linearly), the damping can be found from solutions to the equation

$$\ddot{z} + (const)t^{\frac{1}{2}}z = 0$$

where the constant is dependent on machine parameters. For larger n-values, the amplitude of the oscillations is proportional to $n^{1/2}$, so the damping factor over a region where the n-

value goes from n₁ to n₂ is given roughly by $\left(\frac{n_1}{n_2}\right)^{\frac{1}{2}}$.

We calculated the n-value at a radius r for our data by approximating $\frac{dB_z}{dr}$ with the slope of the line through the data points at radii one step below and one step above r. A plot of the n-value as a function of r for our profile of the magnet with a 30 amp current is shown in Figure 9. We found that our n-value rises linearly in the highlighted region from about 0.5 to 2.8 in. Again appealing to the literature, "The optimum shape of magnetic field in a cyclotron was found to be one in which the n-value rises approximately with radius" (Livingston and Blewett), we conclude that the field created by the new pole pieces has close to the desired property, but may not be optimum.

To get a better idea of how this cyclotron should perform, we made rough comparisons with the MIT 42-in. (21-in. radius) cyclotron. The table below shows the comparison, in which the sizes were roughly scaled around the exit slit location.

Percentage of Exit Slit	n-value	n-value
Distance	(MIT)	(Rutgers)
80	0.02	0.075
100	0.4	0.9
103	3 1	1.19

This shows that the n-value plot for the MIT cyclotron is shallower below and steeper beyond the exit slit than our n-value plot. It also shows, however, that our n-values are quite reasonable, and they may be even better given that this scaling makes the comparison quite rough.

Another interesting result deducible from our profiles is about the saturation properties of the magnet, specifically of the iron cores and the pole pieces. In Figure 5, it is apparent that the increase in maximum magnetic field strength is greater from 20 amps to 30 amps than it is from 30 amps to 40 amps, implying that these currents are already outside of the regime in which the field responds linearly to the magnet current. However, Figure 7

shows that the normalized plots of the radial profile of the magnet field for the three magnet currents are nearly identical. Thus, the shape of the field is unaffected at these currents, so the saturation regime must begin above 40 amps.

We found that the magnetic field created by the new pole pieces was qualitatively just as was desired. The field will induce confinement and focusing and should produce an ion beam within the chamber. More quantitative analysis showed that it should produce a fairly well focused beam. As compared to the MIT cyclotron, the Rutgers 12-in. cyclotron may have stronger axial focusing in the central region and only slightly weaker damping toward the edge of the chamber. (Secondary damping of 28% as compared to 22% for the MIT machine.) The radial focusing may be less efficient toward the center than that of the MIT machine.



Figure 1. The half-circle objects are the dees referred to above, they are open cavities and the particles move within them along the trajectory represented by the dotted lines. If the cyclotron is so designed, there is an opportunity for the beam to leave the chamber as shown. The large grey object is the bottom magnet and is naturally complemented by an additional magnet above the chamber.



Figure 2. This diagram illustrates how an altered pole piece can generate a non-uniform magnetic field, which will provide axial restoring forces due to radial components to the magnetic field that are increasing as you move outwards from the center.



Radius (inches)